Int. J. Heat Mass Transfer. Vol. 23, pp. 897–901
Pergamon Press Ltd. 1980. Printed in Great Britain

HEAT FLOW IN A ROTATING WELD POOL

J. G. ANDREWS and D. A. C. NICOL
Central Electricity Generating Board, Marchwood Engineering Laboratories, Marchwood, Southampton, Hampshire, SO4 4ZB, England

(*Received* 15 *August* 1979 *and in revised form* 17 *December* 1979)

Abstract—A model is proposed for heat flow in a thin plate due to a slowly traversing source in a current-carrying weld pool, which is rotating at high speed due to an applied magnetic field. A simple two-dimensional treatment predicts the distortion of the free boundary between the pool and the surrounding solid. The distortion from a circle depends on the size of a small parameter, ε, which is inversely proportional to the speed of rotation. Significant differences arise between the present model of a vigorously stirred pool and the Rosenthal model of a stagnant pool.

NOMENCLATURE

x, y,	Cartesian co-ordinates;
r, θ,	polar co-ordinates;
$\mathbf{v}$,	velocity field;
u,	traverse speed;
v_θ,	rotational speed;
v_0,	typical rotational speed;
r_0,	radius of pool;
$\hat{\mathbf{x}}, \hat{\boldsymbol{\theta}}$,	unit vectors in x and θ directions;
X, Y, R,	non-dimensional co-ordinates;
ρ,	spatial co-ordinate in boundary layer;
V,	non-dimensional rotational speed;
δ,	δ-function;
L,	latent heat per unit mass;
D,	thermal diffusivity;
k,	thermal conductivity;
c,	specific heat per unit mass;
q,	strength of heat source;
T,	temperature;
T_a,	ambient temperature;
T_m,	melting temperature;
Δ,	Laplacian operator;
Q,	non-dimensional source strength;
ϕ, ϕ_0, ϕ_1,	non-dimensional temperature functions in outer solution;
Φ, Φ_0, Φ_1,	non-dimensional temperature functions in inner solution;
α,	Péclet number based on u;
ε,	inverse of Péclet number based on v_0;
s,	measure of asymmetry;
m,	integer;
I_m, K_m,	modified Bessel functions;
A, B, C, D,	unknown coefficients.

1. INTRODUCTION

THERE is currently considerable interest in welding technology in the application of magnetic fields to stir the pool of molten metal in the vicinity of the heat source [1–3]. The resulting welds are found to have more stable profiles and are less prone to solidification cracking. The existence of vigorous motion in the pool due to self-induced magnetic fields has been known for many years and there have been several attempts to model such effects [4–10]. The point of applying a strong external magnetic field is to overcome the effect of these and any other spurious motions. The associated Péclet number due to rotation of the molten metal is $O(10^2)$ in these experiments. The shape of the pool boundary is observed to be sensitive to the direction and strength of rotation of the pool; in particular, the pool shifts at right angles to the direction of traverse of the source. The main aim of this paper is to predict how the shape of the boundary depends on various parameters.

We consider a two-dimensional model of the weld in a thin plate (see Fig. 1). Current enters the pool at the origin and diverges radially outwards. A magnetic field is applied in the vertical direction which interacts with the current to produce a Lorentz force in the azimuthal direction, causing the fluid to rotate about the origin. In addition, there will also be self-induced motions near the origin where the current enters the material, but we anticipate such effects to be unimportant in thin plates. Heat is generated at the point where the current enters the plate, i.e. at the origin. As we are interested only in the heat flow over distances which are large compared with the plate thickness, it is reasonable to model the heat source as a line source and therefore consider only a two-dimensional problem. The case of a stationary source is of little interest in welding. In most welding processes the source moves relative to the workpiece with constant velocity. Typical traverse speeds are of the order of a few mm s^{-1} which are very small compared with the typical rotational speeds of around 10^2–5×10^2 mm s^{-1} employed by Willgoss and others. We therefore expect significant differences between our model of a rapidly rotating pool and Rosenthal's model [11] in which the liquid motion is ignored. Since we are interested only in explaining the broad qualitative effects, we make the simplifying assumption of an inviscid liquid in the pool. Latent heat is ignored in our solution. An estimate of the effect of latent heat for a moving heat source and a stagnant weld pool has been made by Andrews and Atthey [12]. They conclude that it distorts the melting isotherm by $O(\gamma)$, where $\gamma = L/cT_m$ is the latent heat parameter;

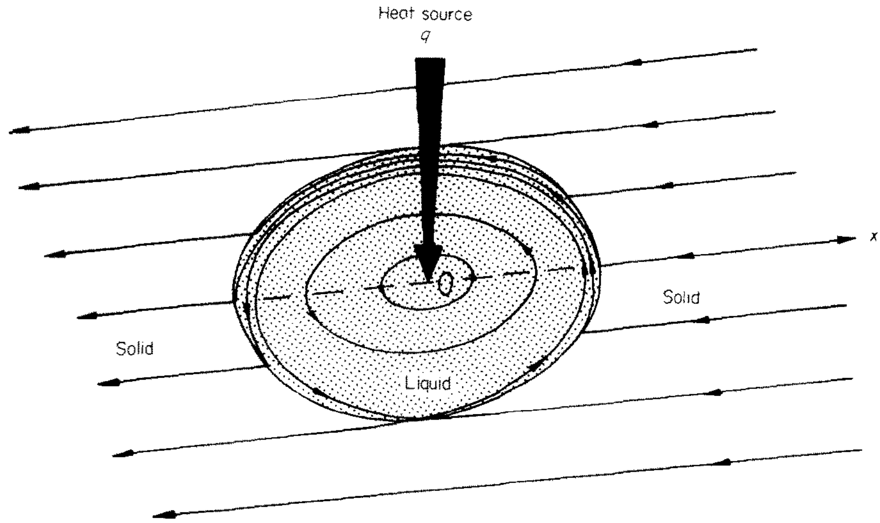

FIG. 1. Streamlines around the pool.

L, c, T_m are the latent heat of fusion, specific heat and melting temperature, respectively. Typically, $\gamma \sim 0.3$ for steels, and we anticipate the distortion due to latent heat to be significantly smaller than that due to the rapid rotation of the pool.

2. MATHEMATICAL MODEL

We consider a point source of heat, q, at the origin, surrounded by a rapidly rotating inviscid liquid pool, which is itself bounded by a solid region streaming slowly in the negative x-direction (see Fig. 1). We assume a velocity field of the form

$$\mathbf{v} = \begin{cases} -u\hat{\mathbf{x}} + v_\theta(r)\hat{\boldsymbol{\theta}}, & \text{in the liquid,} \\ -u\hat{\mathbf{x}}, & \text{in the solid,} \end{cases} \tag{1}$$

and we take the shape of the solid–liquid boundary to be a circle to a first approximation. We anticipate the errors incurred to be only slight provided the predicted distortion of the shape of the melting isotherm from a circle is small. This velocity field implies that there is a trapped region of circulating fluid around the origin and another region near the boundary in which fluid is present for only a finite time. If $|v_\theta| \gg u$ then the region of trapped fluid will occupy the bulk of the pool.

The steady-state temperature field, T, satisfies the heat conduction equation

$$\Delta T = D^{-1}\mathbf{v}\cdot\nabla T - (q/k)\delta(\mathbf{r}) \tag{2}$$

where D, k are the thermal diffusivity, conductivity, respectively (assumed constant), and $\mathbf{v}$ is the velocity at any point in the material. On the solid–liquid boundary the temperature and temperature gradient are both continuous; latent heat effects are ignored in this paper. For convenience, we normalise

$$X = x/r_0, \quad R = r/r_0, \quad \phi = (T - T_a)/(T_m - T_a),$$
$$V = v_\theta/v_0, \quad Q = q/2\pi k T_m, \tag{3}$$

where r_0 is the radius of the pool, T_a is the ambient temperature, T_m is the melting temperature and v_0 is some characteristic rotational speed in the liquid, which is taken to be the fluid speed on the boundary in our inviscid model. We also note the existence of two non-dimensional parameters.

$$\alpha = ur_0/2D, \qquad \varepsilon = D/v_0 r_0. \tag{4}$$

α and ε^{-1} are essentially Péclet numbers for the solid and liquid, respectively. Numerically, $\alpha \sim 1$ and $\varepsilon \sim 10^{-2}$ for typical welding conditions. The normalised form of equation (2) is

$$\phi_{RR} + R^{-1}\phi_R + R^{-2}\phi_{\theta\theta} + 2\alpha\phi_X = \varepsilon^{-1}VR^{-1}\phi_\theta - Q\delta(R), \quad R < 1, \tag{5}$$

$$\phi_{RR} + R^{-1}\phi_R + R^{-2}\phi_{\theta\theta} + 2\alpha\phi_X = 0, \quad R > 1, \tag{6}$$

where the subscripts denote partial derivatives.

The general solution in the solid which decays to zero at infinity is obtained by making the usual substitution $\phi = \exp(-\alpha X)\Psi$ and separating the variables, whence

$$\phi = e^{-\alpha X} \sum_{m=-\infty}^{\infty} A_m K_m(\alpha R) e^{im\theta}. \tag{7}$$

$K_m(\alpha R)$ is the modified Bessel function of the second kind.

In the liquid we look for a simple perturbation solution

$$\phi = \phi_0 + \varepsilon\phi_1 + \cdots \tag{8}$$

where $\phi_0, \phi_1, \ldots$ are periodic functions of θ. Substituting (8) in (5) yields, to $O(\varepsilon^{-1})$,

$$\partial\phi_0/\partial\theta = 0, \tag{9}$$

so that ϕ_0 is a function of R alone. To $O(1)$ we have

$$R^{-1}\frac{d}{dR}\left(R\frac{d\theta_0}{dR}\right) + 2\alpha\cos\theta\frac{d\phi_0}{dR} = VR^{-1}\frac{d\phi_1}{d\theta} - Q\delta(R). \tag{10}$$

Integrating (10) with respect to θ from $\theta = 0$ to 2π and applying the condition that ϕ_1 is periodic we obtain the equation for ϕ_0

$$R^{-1}\frac{d}{dR}\left(R\frac{d\phi_0}{dR}\right) = -Q\delta(R),$$

which has the solution

$$\phi_0 = 1 - Q\ln R, \tag{11}$$

where the constant of integration has been chosen so that the melting isotherm $\phi = 1$ occurs on $R = 1$ to a first approximation. The standard approach would be to match temperature and temperature gradient everywhere around the boundary $R = 1$. However, close inspection of (7), (8) and (11), shows that the form of the perturbation solution precludes continuity of both temperature and temperature gradient everywhere around the boundary. However, we anticipate the azimuthal variation of temperature gradient to be $O(1)$ near the boundary. Hence we look for a thermal boundary layer solution in this region.

From the form of the energy equation (5), we scale

$$\rho = (1-R)\varepsilon^{-\frac{1}{2}}, \quad \Phi(\rho) = \phi(R), \tag{12}$$

so that (5) becomes

$$\Phi_{\rho\rho} - \varepsilon^{\frac{1}{2}}(1 + 2\alpha\cos\theta)\Phi_\rho = \Phi_\theta, \tag{13}$$

on using $V(1) = 1$ and neglecting terms of $O(\varepsilon)$. The appropriate expansion for Φ is

$$\Phi = \Phi_0 + \varepsilon^{\frac{1}{2}}\Phi_1 + \cdots, \tag{14}$$

where Φ_0 satisfies

$$\frac{\partial^2\Phi_0}{\partial\rho^2} = \frac{\partial\Phi_0}{\partial\theta}. \tag{15}$$

Periodic solutions of (15) which can be matched with the outer solution (11) take the form

$$\Phi_0 = B^0 + C^0\rho + \sum_{\substack{m\\ m\neq 0}} D_m^0 \exp\{-e^{sgn(m)i\pi/4}|m|^{\frac{1}{2}}\rho\}e^{im\theta}. \tag{16}$$

Matching (16) as $\rho \to \infty$ with the outer solution (11) as $R \to 1$ gives

$$B^0 = 1 \quad \text{and} \quad C^0 = 0. \tag{17}$$

Before applying the continuity conditions across $R = 1$ we expand the unknown coefficients A_m in (7) as the series

$$A_m = A_m^0 + \varepsilon^{\frac{1}{2}}A_m^1 + \cdots \tag{18}$$

Comparing the radial derivatives of the leading terms in (7) and (14) we observe the coefficients of the D_m^0 terms alone to be $O(\varepsilon^{-\frac{1}{2}})$, so that

$$D_m^0 = 0, \quad \text{for all } m. \tag{19}$$

Equating the temperatures across the boundary to $O(1)$ gives

$$e^{-\alpha\cos\theta}\sum_{m=-\infty}^{\infty} A_m^0 K_m(\alpha)e^{im\theta} = 1, \tag{20}$$

which is readily solved using the result [13]

$$e^{\alpha}\cos\theta = \sum_{m=-\infty}^{\infty} I_m(\alpha)\cos m\theta,$$

to yield

$$A_m^0 = I_m(\alpha)/K_m(\alpha), \tag{21}$$

where $I_m(\alpha)$ is the modified Bessel function of the first kind.

In order to equate the temperature gradients to $O(1)$ we need the next term, Φ_1, in the expansion (14). Noting that $\Phi_0 = 1$, we find that Φ_1 satisfies the same differential equation (15) as Φ_0, with the same general periodic solution (16) (with the superscript 0 replaced by 1). As before, matching to the outer solution in the body of the liquid, yields

$$B^1 = 0, \quad C^1 = Q.$$

Then equating the temperature gradient in the inner region to that in the solid (7) gives

$$-Q + \sum_{\substack{m\\ m\neq 0}} D_m^1 e^{sgn(m)i\pi/4}|m|^{\frac{1}{2}} e^{im\theta}$$
$$= -\alpha\cos\theta + \alpha e^{-\alpha\cos\theta}\sum_m A_m^0 K_m'(\alpha)e^{im\theta}, \tag{22}$$

which we Fourier analyse to find the relationship between Q and α and also derive the coefficients $D_m^1 (m \neq 0)$. Finally, we equate the temperature to $O(\varepsilon^{\frac{1}{2}})$, i.e.

$$e^{-\alpha\cos\theta}\sum_m A_m^1 K_m(\alpha)e^{im\theta} = \sum_{\substack{m\\ m\neq 0}} D_m^1 e^{im\theta} \tag{23}$$

from which we similarly derive the A_m^1 coefficients.

Thus the inclusion of the thermal boundary layer between the body of the pool and the solid allows the construction of an approximate solution valid to $O(\varepsilon^{\frac{1}{2}})$. Furthermore, since the only information from the body of the pool used in the matching is the melting temperature and total heat flux, we observe that the precise distribution of the heat source in the liquid is immaterial, provided it is still radially symmetric.

3. RESULTS

The various infinite series arising from the expressions for temperature and temperature gradient are found to converge very rapidly for typical values of α. Hence the numerical computation of temperature is straightforward. Figure 2 compares the isotherms in the solid region for the present model of a rotating liquid pool in the limit as $\varepsilon \to 0$ with the Rosenthal [11] solution, given by equation (7) with $A_0 = Q$, $A_m = 0$ $(m \neq 0)$. Broadly speaking, the effect of infinite rotation is to create a distributed source over a disc $R = 1$. The upstream isotherms are pushed further upstream and the downstream isotherms are pulled back nearer the source in this case. Figure 3 shows the isotherms for $\varepsilon = 0.02$ and $\alpha = 1$. The upstream isotherms are little changed from the previous case but the downstream isotherms are significantly elongated and asymmetric about the X-axis. This indicates preferential heat flow to one side of the weld, with the boundary of the pool beginning to take the character-

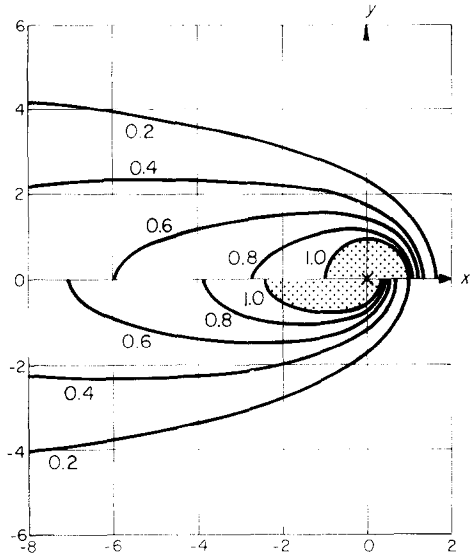

FIG. 2. Comparison between the rotating pool model ($Y \geqslant 0$) for $\varepsilon = 0$ with the Rosenthal model ($Y \leqslant 0$), taking $\alpha = 1$ and $Q = 1.29$.

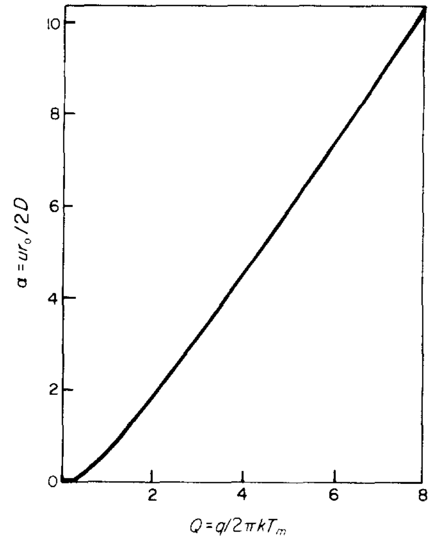

FIG. 4. Relationship between the non-dimensional power, Q, and the traverse Péclet number, α.

istic 'tadpole' shape observed experimentally. Figure 4 shows the relationship between (non-dimensional) power Q, and the traverse Péclet No. α, which is approximately linear for $\alpha > 1$. Finally, Fig. 5 shows the departure of pool shape from symmetry about the X-axis with increasing ε. The measure of asymmetry we adopt is the 'shift factor', s, defined by

$$s = \tfrac{1}{2}\{\min(Y) + \max(Y)\},$$

for points (X, Y) on the melting isotherm $\Phi = 1$; the sign of s depends on the direction of rotation. Unfortunately, there is insufficient experimental data at the present time for the detailed comparison between experiment and theory, though the general trend is in line with preliminary rwsults obtained by Willgoss (private communication). When detailed experimental results are available there would be interest in considering other velocity profiles, in particular those based on a viscous fluid with a rapid variation in velocity near the solid–liquid boundary.

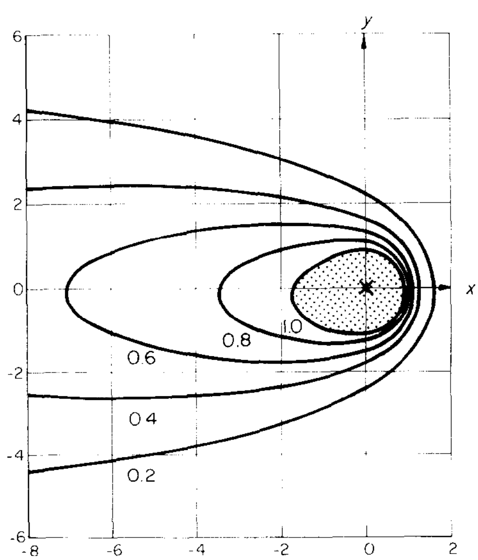

FIG. 3. Isotherms for the case $\varepsilon = 0.02$, $\alpha = 1.0$, $Q = 1.29$. (The shaded region denotes the pool.)

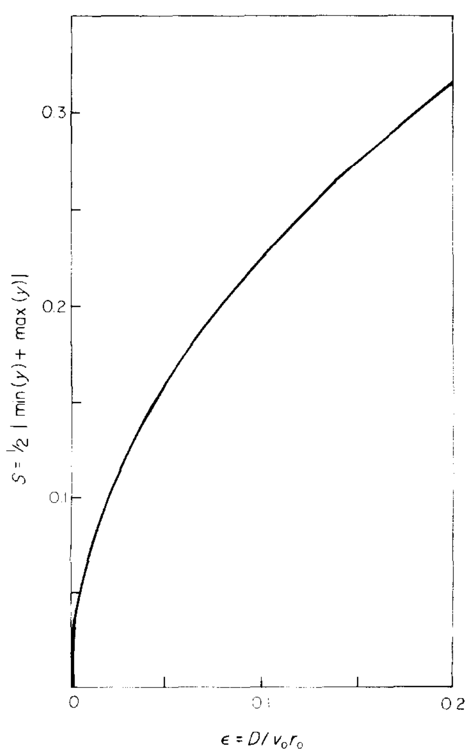

FIG. 5. Plot of the sideways shift of the pool with ε.

4. CONCLUSIONS

The two-dimensional heat flow problem of a slowly traversing source surrounded by a rapidly rotating weld pool can be analysed by means of the small parameter $\varepsilon = D/v_0 r_0$. It accounts for the departure from symmetry of the shape of the pool about the direction of traverse. The results for the rapidly rotating pool differ significantly from those predicted by the Rosenthal model in which there is no fluid motion.

Acknowledgement—This paper is published by permission of the Central Electricity Generating Board.

REFERENCES

1. A. M. Boldyrev, N. P. Tolonnikov and E. B. Dorofeev, Concerning the possibility of obtaining stable reversible motion of the molten metal in the weld pool with electromagnetic stirring, *Welding Prod.* (*USSR*) **22** (11), 4–7 (1975).
2. D. G. Telford and D. R. Milner, Microstructural effects produced by electromagnetic motion, *Welding Res. Int.* **6** (3), 31–34 (1976).
3. W. H. S. Lawson and H. W. Kerr, Fluid motion in GTA weld pools, *Welding Res. Int.* **6** (5), 63–77; (6), 1–17 (1976).
4. R. A. Willgoss, Electromagnetic control of fluid motion in TIG weld pools, *Proceedings of International Conference on Arc Physics and Weld Pool Behaviour*, pp. 361–373, London (1979).
5. J. A. Shercliff, Fluid motions due to an electric current source, *J. Fluid Mech.* **40,** 241–250 (1970).
6. C. Sozou and W. M. Pickering, Magnetohydrodynamic flow due to the discharge of an electric current in a hemispherical container, *J. Fluid Mech.* **73,** 641–650 (1976).
7. C. Sozou and W. M. Pickering, Magnetohydrodynamic flow in a container due to the discharge of an electric current from a finite size electrode, *Proc. R. Soc. Lond.* **A362,** 509–523 (1978).
8. J. G. Andrews and R. E. Craine, Fluid flow in a hemisphere induced by a distributed source of current, *J. Fluid Mech.* **84,** 281–290 (1978).
9. D. R. Atthey, A mathematical model for fluid flow in a weld pool at high current, *J. Fluid Mech.* In press.
10. R. E. Craine and N. P. Weatherill, Fluid flow in a hemispherical container induced by a distributed source of current and a superimposed uniform magnetic field, *J. Fluid Mech.* In press.
11. D. Rosenthal, The theory of moving sources of heat and its application to metal treatments, *Trans. Am. Soc. Mech. Engrs* **68,** 849–866 (1946).
12. J. G. Andrews and D. R. Atthey, The response of a weld pool to perturbations in power, *Int. J. Heat Mass Transfer* **22,** 1533–1538 (1979).
13. M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, Applied Maths. Series, Vol. 55, p. 376 (1964).

FLUX THERMIQUE DANS UNE SOUDURE ANIMEE D'UNE ROTATION

Résumé—On propose un modèle pour le flux thermique dans une plaque mince, du à une source la traversant lentement dans une soudure porteuse de courant qui tourne à grande vitesse sous l'effet d'un champ magnétique. Un traitement simple, bidimensionnel prévoit la distorsion de la frontière entre le bain et le solide. La distorsion par rapport à un cercle dépend de la valeur d'un petit paramètre ε qui est inversement proportionnel à la vitesse de rotation. Des différences sensibles apparaissent entre ce modèle d'un bain liquide vigoureusement brassé et celui de Rosenthal d'un bain au repos.

WÄRMEFLUSS IN EINEM ROTIERENDEN SCHMELZBAD

Zusammenfassung—Für den Wärmefluß in einer dünnen Platte, der von einer langsam querbewegten Wärmequelle in einem stromdurchflossenen Schmelzbad erzeugt wird, welches sich infolge eines magnetischen Feldes mit hoher Geschwindigkeit dreht, wird ein Modell vorgeschlagen. Durch eine einfache zweidimensionale Behandlung läßt sich der Verlauf der Phasengrenze zwischen dem festen und flüssigen Werkstoff berechnen. Die von einem Kreis abweichende Form des Schmelzbades hängt von der Größe des kleinen Parameters ε ab, welcher umgekehrt proportional zur Rotationsgeschwindigkeit ist. Bedeutende Unterschiede ergeben sich zwischen diesem Modell eines stark gerührten Bades und dem Rosenthal-Modell für ein ruhendes Bad.

ТЕПЛОВОЙ ПОТОК ВО ВРАЩАЮЩЕМСЯ ОБЪЕМЕ РАСПЛАВА

Аннотация — Предложена модель теплового потока к тонкой пластине, генерируемого источником тепла, медленно перемещающимся в токопроводящем объеме расплава, который вращается с большой скоростью под действием приложенного магнитного поля. Простое исследование двумерной модели позволяет определить возмущение свободной границы между расплавом и окружающим твердым телом. Отклонение границы от формы круга зависит от величины малого параметра ε, обратно пропорционального скорости вращения. Отмечены существенные различия между предложенной моделью интенсивно перемешиваемого объема расплава и моделью Розенталя для неподвижного объема жидкости.